Slot Theory and Slotite Theory

ABSTRACT: Slot theorists believe that properties/relations have slots which are filled by instances/relata. One problem this theory faces is 'The Problem of Filling'. This paper examines a variety of solutions, one of which is an extension of slot theory which adds an additional category of entities, 'slotites'.

1. Introduction

1.1 Slot Theory

Slot theorists assume that, in addition to properties/relations existing, there exist 'slots' (Dixon [2018b: 5615] lists those who endorse it; Gilmore [2013] and Dixon [2018a] defend it in-depth). As a, somewhat crass, first pass understanding, imagine the slot theorist says that every property/relation possesses a number of slots equal to its adicity e.g., using Italics to represent the names of properties, Red has one slot and Married has two slots. These slots are then filled by different relata of the relation (or instances of the property). For instance, Red's slot might be filled by a red ball which instantiates Red whilst Married's slots would be filled by pairs of spouses. This crass statement of slot theory would have the following three pieces of (possibly primitive, possibly non-primitive) ideology:

1. (CSs) ‘__ is a slot’ (which holds of slots).
2. (CSp) ‘__ is possessed by ___’ (the first relatum is a slot; the second relatum is a property/relation).
3. (CSf) ‘__ fills ___’ (the first relatum is a relatum/instance of a relation/property; the second relatum is a slot possessed by that relation/property).

Furthermore, there’s obviously some connection between instantiation and slot-filling. This first pass, crude version of slot theory says that if some $x$ fills a slot possessed by Property then $x$ instantiates Property. For instance, in the case of Taller Than—which has two slots, $\sigma_1$ and $\sigma_2$—we end up with the following bi-conditional:

$\equiv_{\text{CRUDE}}$: For any $x$ and $y$, $x$ and $y$ jointly instantiate Taller Than—and $x$ is thereby taller than $y$—iff $x$ fills $\sigma_1$ and $y$ fills $\sigma_2$.

1.2 The Problem of Filling (POF)

$\equiv_{\text{CRUDE}}$ is too crude (Dixon [2018a: 201-04] also notes the following problem). Imagine: Abigail is taller than Bronia and both are fully grown women of a tall stature; Clare is taller than Damaris, but both are young children who are both short comparable to Abigail and Bronia i.e. neither Clare

1 Acknowledgements: Many thanks to the comments and feedback at the Madrid Universals Conference 2019. And thanks to Javier Cumpa for inviting me to it!
nor Damaris are taller than Abigail or Bronia. Given filling is a dyadic relation between a relation’s relatum and one of its slots: Abigail fills $\sigma_1$ whilst Bronia fills $\sigma_2$ (for Abigail is taller than Bronia); Clare fills $\sigma_1$ and Damaris fills $\sigma_2$ (for Clare is taller than Damaris); but then, given $\equiv_{\text{CRUD}}$, Clare is thereby taller than Bronia. Since that’s false, we have a problem. Call this the Problem of Filling (POF for short).

This paper discusses how slot theorists might try and solve POF, using it as a chance to survey different slot theories. §§2-4 discuss slot theories which relativise filling to some further entity: instantiations (§2); sums, pluralities, or ordered tuples (§3); and spacetime regions (§4). I then discuss an extension which adds in an extra ontological category to help—‘slotites’ (§5). It not only solves POF but also a problem with symmetric relations. §§6-8 discuss different varieties of slotite theory. Note that this paper does not argue for a specific theory; whilst some have clear downsides, the remainder all have their charms. Rather, this paper is an exploration of logical space, surveying the different theories available and seeing the different metaphysical itches which they can scratch.

2. Instantiation Relative Slot Theory

Roughly speaking, Dixon’s suggestion [2018e: 204] is to relativize filling to instantiations. Call that ‘Instantiation Relative Slot Theory’. Treating instantiations as states of affairs, we replace (CSf) with:

\[
(\text{IRf}) \quad \text{‘}_1 \text{ fills } _2 \text{ relative to } _3\text{’} \quad \text{(the first relatum is a relatum-instance of a relation/property; the second relatum is a slot possessed by that relation/property; the third relatum is a state of affairs with constituents including both that relatum and the relation/property).}
\]

Thus, Abigail and Bronia fill $\sigma_1$ and $\sigma_2$ relative to their instantiation of the Taller Than relation, whilst Clare and Damaris fill those slots relative to their instantiation; there is also an instantiation involving Bronia and Clare, but relative to that instantiation Bronia fills $\sigma_1$ and it is Clare who fills $\sigma_2$. Problem solved. Instead of $\equiv_{\text{CRUD}}$, we’d instead endorse:

\[
\equiv_{\text{INSTR}}. \text{ For any } x \text{ and } y, x \text{ and } y \text{ jointly instantiate Taller Than iff there exists an instantiation, } \theta, \text{ and both } (i) \text{ x fills } \sigma_1 \text{ relative to } \theta, \text{ and (ii) y fills } \sigma_2 \text{ relative to } \theta.
\]

We might want to avoid Instantiation Relative Slot Theory. To see why, first we must better understand the relationship between filling and instantiation. Whilst slot theorists will agree that there’s a bi-conditional connection between filling and instantiation (e.g. $\equiv_{\text{INSTR}}$ or $\equiv_{\text{CRUD}}$), they’ll vary over which side of the bi-conditional is more fundamental than the other. There are three types of fact: facts about which slots are filled (‘filling facts’); facts about which instances/relata instantiate which property/relation (‘instantiation facts’); facts about what the instances/relata are like (e.g. the ball being red or Jack and Jim being married; call them ‘predicational facts’). We then end up with three different views over how these facts relate to the bi-conditional: FILLING FIRST is the principle that filling facts ground instantiation and predicational facts; INSTANTIATION FIRST is the principle that instantiation facts ground filling and predicational facts; PREDICATION FIRST is the principle that predicational facts ground filling and instantiation facts.2 (I ignore the prima facie implausible idea that these facts are independent of one another.) Which of FILLING FIRST,

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2 I talk about ‘grounding’, ‘priority’, and ‘fundamentality’ with some measure of abandon. Feel free to adopt your metametaphysics of choice and parse what I say into your favoured lingo.
Imagine a place many of us can identify, where the respective entities involved. That said, and purely in the interests of metaphysical exploration, it is not. Filling facts are more fine-grained than the instantiation facts. To discover that Abigail and Bronia jointly instantiate Taller than is not as informative as finding out who fills which slot (since it’s that fact which determines who is taller than who). This may not be a reason to prefer Filling First to Predication First, since predication facts are likewise fine-grained, but it’s clearly a reason to prefer Filling First to Instantiation First. And since Instantiation First seems prima facie plausible, Filling First must do as well and we can see the reason to think Filling First is true. That said, and purely in the interests of metaphysical exploration, §§3-4 assume that Filling First is true and try to find alternative solutions to POF which salvage it.

3. Entity Relative Slot Theory

The next two sections consider theories that solve POF by relativizing to entities other than instantiations. One alternative, suggested to me by Donnchadh O’Conaill, is to relativise filling to the respective entities involved. For single slot properties, there’s no problem e.g. the red ball fills Red’s single slot relative to the red ball whilst the red pen fills Red’s slot relative to the red pen. Things become problematic with polyadic relations. Relative to what exactly do their relata fill their slots? A natural answer would be the mereological sum of the relata. But imagine Rness is a two-place many-one relation corresponding to a relational predicate R. Rness has two slots, \( \sigma^1 \) and \( \sigma^2 \). Imagine \( a, b, c \) exist, such that:

(a) \( a \) and \( b \) stand in \( R \) to \( c \),
Having relativized filling to mereological sums, we should endorse:

\[ \text{\textit{First Reduction:}} \quad \text{For any } x \text{ (or } y) \text{ and } y, \text{ } xRy \text{ (or } xSy) \text{ iff (i) } x \text{ (or the } y) \text{ fills } \sigma^1_a \text{ relative to the mereological sum (composed of the } x \text{ plus } y) \text{ and (ii) } y \text{ fills } \sigma^2_a \text{ relative to that same sum.} \]

Given (a) and \( \equiv_{\text{EnRel}} \), \( \epsilon \) fills \( \sigma^2_a \) relative to the mereological sum \( a+b+c \). Given (b) and \( \equiv_{\text{EnRel}} \), the plurality of \( b \) and \( c \) fill \( \sigma^1_a \) relative to \( a+b+c \). Given \( \equiv_{\text{EnRel}} \), the plurality of \( b \) and \( c \) are \( R \)-related to \( \epsilon \) which, given (c), is false. So mereological sums can’t play the relativizing role. (And the same problem applies to relativising to pluralities of the requisite relata e.g. \( \epsilon \) filling \( \sigma^2_a \) relative to the plurality of \( a, b \) and \( c \)

The problem comes about because mereological sums are blind to the ‘order’ of their constituents (and pluralities are blind to the order of the things amongst them). Some entities instead have a metaphysical structure better suited to capture that order. States of affairs are an obvious example, but if we start using them we’ll end up back at Instantiation Relative Slot Theory. The obvious alternative would be to use ordered tuples. Given (a), \( a \) and \( b \) fill \( \sigma^0_a \) relative to \[ \{a,b\}, \varnothing \] and \( \epsilon \) fills \( \sigma^2_a \) relative to \[ \{a,b\}, \varnothing \]. Given (b), \( b \) and \( c \) fill \( \sigma^1_a \) relative to \[ \{b,c\}, \varnothing \] and \( a \) fills \( \sigma^2_a \) relative to \[ \{b,c\}, \varnothing \]. Unlike with mereological sums, the entities playing the relativizing role (i.e. \( \{a,b\}, \varnothing \) and \( \{b,c\}, \varnothing \)) aren’t identical and thus we can’t generate the problem we had with sums and pluralities.

I don’t deny that relativising to ordered tuples is an open option. However, having brought ordered tuples into the picture we might want to pause as to why we don’t go the whole hog and endorse class nominalism. Is the complaint that class nominalism requires classes to exist? Because the same is now true of Entity Relative Slot Theory. Is it that it requires arbitrarily identifying tuples with sets [Benacerraf 1965]? The same problem faces Entity Relative Slot Theory. And so on.

Having already roped in ordered pairs, class nominalism should be a natural theory to endorse.

One issue with this is that Gilmore explicitly ties slot theory to Platonic universals [Gilmore 2013: 187]. But there’s no reason to build this into slot theory. Why not think that other entities, e.g. immanent universals, mereological fusions, or classes, might have slots? Just as the Platonist introduces slots as \textit{sui generis} entities standing in a primitive ‘possession’ relationship to a property, we can say the same for these other theories. With little said about what ‘possession’ amounts to (because there’s not even a rudimentary analysis of it) there’s no impediment to saying that these things can ‘possess’ \textit{sui generis} slots (see also Effingham [Forthcoming]).

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3 This footnote dashes the hopes that class nominalists can reduce slots to some other kind of set-theoretic entities. Imagine four objects, \( a-d \), and two relations, \( R_{ax} \), \( R_{ax}^* \), and \( R_{ax} \) (which correspond to the relational predicates \( \sim R_{ax} \), \( \sim R_{ax}^* \), and \( \sim R_{ax}^\prime \)). Assume: \( Rab, Rbd, R_{ax}^*ab, R_{ax}^*cb, R_{ax}ab, \) and \( R_{ax}cb \) are all and only the true facts. It follows that:

\[ \text{\textit{Rax}} = \{\langle a, b \rangle, \langle b, a \rangle \} \quad \text{\textit{Rax}}^* = \{\langle a, b \rangle, \langle c, b \rangle \} \quad \text{\textit{Rax}}^\prime = \{\langle a, b \rangle, \langle d, b \rangle \} \]

The underlining represents which ‘bit of the set’ corresponds to the ‘first slot’. I can think of only two reductions of slots to sets. Neither works; at least, not if we assume slots theorists want to rule out relations ‘sharing’ slots.

\textit{First Reduction:} Slots are ordered tuples consisting of the underlined element corresponding to the slot. In that case the first slot of \( R\textit{ax} \) would be \( \{\langle a \rangle, \langle b \rangle \} \). But applying the same thinking to \( R\textit{ax}^* \) we get that \textit{the same set} is the first slot of \( R\textit{ax}^* \). \textit{Second Reduction:} Assume a Lockean principle of abstraction which ‘abstracts away’ selected urelements from that set, replacing the appropriate relata with unbound variables (just as open sentences replace names in sentences with unbound variables). The first slot of \( R\textit{ax} \) would be identical to \( \{\langle a \rangle, \langle b \rangle \} \). But we get the same results for \( R\textit{ax}^* \).
But we need more than class nominalism being consistent with slot theory. For our purposes here, we need Filling First to also be true. And I think that’s a step too far. Stereotypical class nominalists (e.g. Lewis [1986: 50-69]) appear to endorse Predication First. Another reading of stereotypical class nominalism is that instances/relata instantiate properties/relations in virtue of being members of an ordered tuple that’s a member of that property/relation. Since facts about membership are grounded solely in facts about what exists in tandem with the axioms of set theory, there’d be just no room for filling facts to play a role in explaining instantiation facts i.e. Filling First must be false given such a reading of class nominalism.

Overall, I don’t deny that there’s room in logical space for relativising filling to ordered tuples. But, as this section has explained, it does raise awkward questions about the overall theory we’re left with: Given there are ordered tuples playing a crucial role in our metaphysical scheme, why not be a class nominalist? And if you’re a class nominalist, how do you square that with Filling First?

4. Spatiotemporal Relative Slot Theory

Spacetime is a dimension along which we commonly find variation of facts. I am tall now, but short then; the kilt is red in one place, green in another; it is raining in Birmingham, but not in Madrid. So it’s natural to relativise filling facts to regions of spacetime—a slot may be filled by something in one place but unfilled in another, just as whether ‘here’ is occupied by matter may vary as you move across space. Call this ‘Spatiotemporal Relative Slot Theory’. It’s natural to add that slots would themselves be spatiotemporally located, presumably occupying wherever the relevant relata are (and, equally, it’d be natural to assume that properties are likewise located, either being immanent universals or mereological fusions). We replace (CSf)/(IRf) with:

(SRf) ‘__ fills __ relative to ___’ (the first relatum is a relatum/instance of a relation/property; the second relatum is a slot possessed by that relation/property; the third relatum is a spacetime region).

And then endorse:

=STR. For any x and y, x and y jointly instantiate Taller Than iff there exists some spacetime region, α, and both (i) x fills σ₁ relative to α, and (ii) y fills σ₂ relative to α.

(Filling is region-relative, whilst instantiating the property is not; this is as it should be.)

Reconsider POF’s problematic example case in light of =STR. Imagine Abigail, Bronia, Clare, and Damaris respectively exactly occupy four regions, rₐ, rₘ, rₜ, and rₕ. Abigail fills σ₁ relative to rₐ∪rₘ; Bronia fills σ₂ relative to rₜ∪rₕ; Clare fills σ₁ relative to rₘ∪rₜ; Damaris fills σ₂ relative to rₕ∪rₘ; Bronia fills σ₁ relative to rₕ∪rₘ; whilst Clare fills σ₂ relative to rₘ∪rₕ; and so on. Because the thing we relativise to is appropriately different each time, POF appears to have been solved.

But spacetime relativisations are always an awkward beast since it’s possible that there exist superposed entities e.g. statues and lumps of clay, ghosts and material objects, or superposed bosons. Imagine that four bosons exist, b₁-b₄; b₁ and b₄ spin whilst b₂ and b₃ do not; b₁ and b₂ are superposed (and exactly occupy r₁); b₃ and b₄ are superposed (and exactly occupy r₂). Bosons are spinlike, and jointly instantiate the relation Spinlike, iff they have the same spin. Thus b₁ and b₄ jointly instantiate Spinlike, as do b₂ and b₃. σ₁ and σ₂ are the two slots which Spinlike possesses. Thus: b₁ fills σ₁ relative to r₁∪r₂ (because b₁ has the same spin as b₄) whilst b₄ fills σ₂ relative to
When it comes to POF at we know which slotites ‘hook each other’ and which of temporary slotite pairs are yoked together so that Abigail, Bronia, Clare, and Damaris, Bronia and Clare, and Bronia and Damaris are another pair, slotites and instances.

To get a different style of solution, we should compare POF to the problem of temporary intrinsics [Wasserman 2006]. Roughly speaking, there are two styles of response to that problem: (i) endurantists relativise instantiation to times (or spacetime points, or utilise time-relativised adverbs etc.); (ii) perdurantists, rather than relativising instantiation, add in extra ontology (i.e. temporal parts) to do the work. When it comes to POF I’ve discussed options which relativise filling; those responses are analogous to endurantist responses to the problem of temporary intrinsics. It’s not hard, though, to imagine a response analogous to perdurantism. We add in extra ontology, ‘slotites’, to solve POF. Just as the perdurantist demands a plethora of temporal parts (one for every instant that an object exists at), slotite theorists commit to a plethora of slotites. As a first pass, imagine that every property/relation has a number of slotites equal to its adicity multiplied by the number of its instantiations, such that each instantiation gets its own, distinct slotite. For example, returning to Abigail, Bronia, Clare, and Damaris, there are six instantiations of Taller than (Abigail being taller than Bronia, Bronia being taller than Clare, Abigail being taller than Damaris etc.). Since Taller than is dyadic, there must then be twelve slotites, τ₁, τ₂…τ₁₂. Having used ‘filling’ for the relation between slots and instances, use ‘plug’ for the similar relation between slotites and instances. One pair of slotites, τ₁ and τ₂, are plugged by Abigail and Bronia respectively; another pair, τ₃ and τ₄, are plugged by Clare and Damaris respectively; similarly, for Abigail and Clare, Abigail and Damaris, Bronia and Clare, and Bronia and Damaris. The slotite theorist then adds that these pairs are ‘yoked together’ so that we know which slotites ‘hook up’ with which other slotites. Thus τ₁ and τ₂ are yoked, τ₁ and τ₄ are yoked, τ₃ and τ₆ are yoked etc. and no other pairs are yoked i.e. τ₁ and τ₃ aren’t yoked, nor τ₁ and τ₄, nor τ₁ and τ₅ etc.

(Technically, Abigail and Bronia don’t really plug slotites, for their temporal parts do instead. For any two (possibly identical) times, t and t′, such that Abigail exists at t and Bronia at t′, there’s a pairing of Abigail’s t-temporal part, aₜ, and Bronia’s t′-temporal part, bₜ′. For each pairing: distinct slotites exist, τ₁ and τ₂; aₜ plugs τ₁; bₜ′ plugs τ₂; τ₁ and τ₂ are yoked. For ease of presentation, I’ll ignore this complication about temporal parts.)

Slotite theory needs different ideology than slot theory:

(STi) ‘__ is a slotite’ (which holds of slotites).

(STp) ‘__ is possessed by __’ (the first relatum is a slotite; the second relatum is a property/relation).

(STh) ‘__ plugs __’ (the first relatum is a relatum/instance of a relation/property; the second relatum is a slotite possessed by that relation/property).

(STy) ‘__ is yoked to__’ (both relata are slotites).
Regarding *Taller than*, slotite theorists say:

\[ \text{slots} \text{ theorists, } x, y \text{ and } z \text{ jointly instantiate } Taller \text{ iff there exist two slotites, } \tau_x \text{ and } \tau_y \text{ such that: (i) } x \text{ plugs } \tau_x \text{ (ii) } y \text{ plugs } \tau_y \text{ (iii) } \tau_y \text{ are yoked; (iv) } \tau_x \text{ and } \tau_y \text{ are both possessed by } Taller. \]

\[ \text{slots} \text{ theorists, } x, y \text{ and } z \text{ jointly instantiate } Taller \text{ iff there exist two slotites, } \tau_x \text{ and } \tau_y \text{ such that: (i) } x \text{ plugs } \tau_x \text{ (ii) } y \text{ plugs } \tau_y \text{ (iii) } \tau_y \text{ are yoked; (iv) } \tau_x \text{ and } \tau_y \text{ are both possessed by } Taller. \]

\[ \text{slots} \text{ plus the facts about which slotites are yoked, solves POF. It can also avoid the problem of superposed objects. Since } b_1 \text{ and } b_4 \text{ are spinlike-related, they have their own pairing of yoked slotites which they plug; } b_2 \text{ and } b_3 \text{ are likewise. When it comes to } b_1 \text{ and } b_2 \text{ the slotite theorist will either say that, because they aren’t spinlike-related, either there isn’t a pair of slotites or (if the slotite theorist is willing to add in more slotites) there is a pair of yoked slotites, but they go unplugged. Whichever is the case, } \text{slots} \text{ entails that } b_1 \text{ and } b_4 \text{ are spinlike-related, as are } b_2 \text{ and } b_3. \]

But } b_1 \text{ and } b_4 \text{ are not. Problem solved!}

5.2 Objections to Slotite Theory

*Objection One:* Slotite theory demands an increase in one’s ontology, for now there are a lot of slotites. But, just as perdurantism’s extra ontological commitments aren’t normally thought to be a deal breaker, we should say the same of slotite theory.

*Objection Two:* One may complain that, whilst slots are an intuitive category of entity, slotites are weird, contrived, and/or otiose. But we should, again, compare the theory to perdurantism. Just as perdurantism ‘uncharts’ a commitment to temporal parts that were originally thought to be a bizarre category of entity [Chisholm 1976: 143; Geach 1972: 311; van Inwagen 1981: 133], slotite theorists should treat their theory as having made an ontological discovery. Just as people came to grips with temporal parts, I’m sure that if we spent long enough trucking in slotites rather than slots they’d become quite natural items of one’s ontology.

*Objection Three:* Slotites must come in different ‘flavours’. Consider the pair of yoked slotites, \( \tau_1 \) and \( \tau_2 \), plugged by Abigail and Bronia respectively. There must be something *about* these slotites which means that Abigail plugging \( \tau_1 \) and Bronia plugging \( \tau_2 \) makes the former taller than the latter; \( \tau_1 \) must be a ‘tall-flavoured’ slotite whilst \( \tau_2 \) must be a ‘short-flavoured’ slotite. But what explains these flavours?

Such concerns are dialectically inappropriate, because we’re comparing slotite theory to slot theories and their variants. Even the most vanilla slot theorist will already have something to say about such ‘flavouring’, saying: *Taller than* possesses two slots; Abigail fills one slot; Bronia fills the other; something about these slots means that what fills the first slot is the thing which is tall compared to the shorter thing which fills the other slot. But in that case, slots also have ‘flavours’. (See also Fine’s [2000: 10] diagrammatic representation of positionalism, which represents the slots differently—that is, as having different flavours—purposefully capturing exactly this idea.) And if slots have flavours, why not slotites? Alternatively, the slot theorist may roll out some reason why they don’t need slots to have flavours—but then I expect that the same reasoning will apply to slotites!

It’s doubly dialectically inappropriate if we’re realists who deny *Predication First*. Then a ball is red in virtue of it instantiating *red*. So either something about *red* explains why its instances are red (in which case properties come in different ‘flavours’—why not, then, slotites?) or nothing does (and, again, why can’t we then say slotites don’t need flavours either?).
Objection Four: We’ve stopped talking about slots and so slotite theory isn’t really a variant of slot theory. But slots can be easily recovered. Just as temporal parts compose objects, slotites could compose slots. Assuming that both slotites and slots come in flavours, imagine that slots are fusions of all and only the slotites of the same flavour e.g. one slot of Taller than is composed of the first slotite of every pair of yoked slotites, whilst the other slot is composed of the remainder. (We can further analyse filling slots in terms of plugging slotites that are parts of the slot; similarly, we can say that a property possesses a slot iff it possesses the slotites composing the slot.)

Objection Five: Slotite theory threatens believing in properties in the first place. All of the heavy lifting in our metaphysical theory is being carried out by things plugging slotites of certain flavours. We need no longer explain why red things are red in terms of their instantiating Red when we can explain their being red in terms of plugging a certain flavour of slotite.

One option is to admit this and drop properties in favour of slotites. Another option is to recover properties just like we recovered slots, saying that properties are composed by the slotites that they possess. Properties would be derivative entities, much like a table is derivative of its atoms. And just as the derivative nature of tables is no slight against the existence of tables, the derivative nature of properties would be no slight against properties.

Objection Six: One may worry that slotites are just tropes in all but name. But this isn’t the case. Admittedly, in monadic cases they seem the same. Consider some red balls. The trope theorist says that, to each ball, there exists a distinct particular; a distinct red trope. The slotite theorist says that, to each ball, there exists a distinct particular; a distinct slotite. But in polyadic cases, the similarity ends. If the trope theorist admits of relational tropes, then two spinlike bosons are related by a single spinlike trope. But the slotite theorist says there are two things i.e. the two yoked slotites. Demonstrably, slotites and tropes are different.

What I don’t deny is that we might be able to build up connections between tropes and slotites. Just as properties and slots were identified with fusions of slotites, we can say the same of tropes—slotites are the distinct ‘legs’ of a relational trope and tropes are ‘built up’ out of slotites. An unyoked slotite, e.g. the slotite possessed by Red and plugged by a ball, is a trope. Similarly, a fusion of maximally yoked slotites is a trope. So tropes are ontologically reducible to slotites.

5.3. The Problem of Symmetric Relations

Slotite theory solves POF. It also solves Fine’s [2000: 17-22] problem for slot theory when it comes to symmetric relations. Non-symmetric relations have slots with different flavours and it’s clear which relatum fills which slot. But with symmetric relations things are problematic. Spinlike has two slots, $\sigma_1$ and $\sigma_2$; spinlike-related bosons, like $b_1$ and $b_4$, fill those slots. But either slot is as

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4 It’s a bit odd to say properties ‘possess’ slotites when the slotites are ontologically prior to the property. Better to say that properties are composed of ‘property forming’ slotites. We can then analyse possession in terms of that. Assume that, for every different flavour of slotite, there’s a primitive monadic predicate applying to the slotite e.g. ‘__ is flavour 1’, ‘__ is flavour 2’ etc. (This assumption could be dispensable, depending upon how we treat the issue of flavours of slotites/properties.) The relation ‘__ has the same flavour as __’ is definable in terms of those predicates. Further, we can define a disjunctive relation ‘__ either has the same flavour as __, or is yoked to it’. Call the ancestral of that relation $\Theta$. Define:

- The $x$s are property forming $=_{df}$ The $x$s are slotites which are $\Theta$-related to one another and nothing which is $\Theta$-related to any $x$ fails to be amongst the $x$s.
- $x$ is a property $=_{df}$ $x$ is composed out of property forming slotites.

$\text{Fness}$ possesses slotite $\tau =_{df} \tau$ is amongst the slotites which compose $\text{Fness}$. 

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good as the other! Perhaps (i) \( b_1 \) fills \( \sigma_1 \) and \( b_4 \) fills \( \sigma_2 \) but equally we might say (ii) \( b_1 \) fills \( \sigma_2 \) and \( b_4 \) fills \( \sigma_1 \). In (i) and (ii) different slots are being filled by different things, thus they’re different states of affairs. But clearly (i) and (ii) are the same state of affairs i.e. the state of affairs of \( b_1 \) and \( b_4 \) being spinlike. Since it’s contradictory for states of affairs to be both distinct and the same, slot theory has a problem.

Dixon’s solution [2018a] is that symmetrical relations have just a single slot filled by a plurality of entities e.g. Spinlike has a single slot filled by the plurality of \( b_1 \) and \( b_4 \) (where that filling would be relativised to an instantiation, or what have you). Given there’s a single slot, there’s only ever one state of affairs—the problem is solved. But Dixon’s solution is revisionary, making Spinlike monadic when it’s intuitively dyadic. This occludes the prima facie difference between monadic properties instantiated by pluralities and symmetric polyadic relations. To better understand, first distinguish the two ways to understand ‘adicity’. We might mean the number of places a relation has or we might mean the number of things which can fill those places [cf Morton 1975: 309-10; Oliver and Smiley 2004: 615-18]. For instance, the property Forming a circle—as in ‘Those points form a circle’—appears to have one place but that place can be filled by a varying number of people. Stipulate that ‘adicity’ refers to places. So Forming a circle is a monadic collective predicate. Intuitively, this seems different from Spinlike which appears to relate two entities in so far as the relation has two places—intuitively, it’s dyadic. Dixon’s theory makes Spinlike akin to Forming a circle; it ends up being monadic when, intuitively, it’s dyadic.

Slotites avoid Fine’s problem whilst keeping Spinlike dyadic. Assume that slotites are like tropes in that, just as a trope is necessarily had by whatever has it (and, e.g., my tropes can’t be shared with anyone else), slotites are only ever plugged by whatever actually plugs them. Constructing a Finean problem case is now impossible. There’s only one state of affairs, namely \( b_1 \) plugging slotite \( \tau'_1 \) and \( b_4 \) plugging slotite \( \tau'_2 \). Given the assumption, it’s now impossible for the reverse state of affairs to hold i.e. for \( b_1 \) to plug slotite \( \tau'_2 \) and \( b_4 \) to plug slotite \( \tau'_1 \).

We can now correctly capture the adicity of Spinlike:

A property/relation, \( F_{nexit} \), is \( n \)-adic \( \equiv \) There are \( n \) slotites, the \( \tau \)s, such that (i) each is possessed by \( F_{nexit} \) and (ii) the \( \tau \)s are yoked together.

For example, a symmetrical relation like Spinlike is such that it possesses pairs of slotites i.e. always two slotites yoked together. Thus, Spinlike is dyadic.

5.4 Dimensions of Variation

The above move requires a necessary connection between slotites and those things which plug them. Necessary connections are usually thought to be mysterious. One way to resolve such mystery is to say that the connected entities stand in some sort of relation of ontological dependence; that is, either the things which plug the slotites depend upon the slotites or the slotites depend upon the things which plug them. That’s one dimension that slotite theories may differ over.

Another dimension of variation concerns one’s take on Filling First/Instantiation First/Predication First. We now get an additional position: Facts about what slotites are plugged explain the instantiation facts, predicational facts, and facts about which slots are filled. Call that Plugging First. Once you take on-board the existence of slotites it seems bizarre to now think that filling slots does the explaining—if you’re attracted to Filling First you should now
instead be attracted to Plugging First. That leaves slotite theories varying over which of Instantiation First, Predication First, and Plugging First is true.

These two dimensions in mind, we can carve up the slotite theories into the following:

<table>
<thead>
<tr>
<th>Base</th>
<th>Dependence of...</th>
<th>Slotites on instances</th>
<th>Instances on slotites</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plugging First</td>
<td></td>
<td>Byzantine Slotite Theory</td>
<td></td>
</tr>
<tr>
<td>Instantiation First</td>
<td></td>
<td>Grounded Slotite Theory</td>
<td></td>
</tr>
<tr>
<td>Predication First</td>
<td></td>
<td>Nominalist Slotite Theory</td>
<td></td>
</tr>
</tbody>
</table>

(There may be more dimensions of variation than these two. For instance, can there exist unplugged slotites? Or is every slotite plugged? Or we could ask whether slotites depended upon properties or properties upon slotites? But I will limit my investigation only to these two dimensions.)

6. Nominalist and Grounded Slotite Theory

Just as slot theorists may think that instantiation facts are the explanatory base for both filling and predicational facts, slotite theorists might think likewise of plugging facts. Call this ‘Grounded Slotite Theory’. You won’t be a grounded slotite theorist because of anything to do with POF. Having endorsed Instantiation First, Instantiation Relative Slot Theory will work fine in solving POF—no slotites required! But slotites will still be useful in solving the problem of symmetric relations. Moreover, one might be quite permissive about ontology [Schaffer 2009: 356-62] and so be committed to slotites anyhow. So there are reasons to think that those who endorse Instantiation First will nonetheless endorse slotite theory and be grounded slotite theorists.

Grounded slotite theory is worth noting. But, just as I’ve focussed on Filling First for most of the paper, I want to focus on its slotite-equivalent, Plugging First. So I’ll set aside grounded slotite theory. Similarly, a slotite theory that accept Predication First (‘Nominalist slotite theory’, because the world is nominalistic at the fundamental level) may be deserving of a proper investigation, but in the context of this paper I will move on from it to discuss slotite theories accepting Plugging First.

7. Byzantine Slotite Theory

It is natural to pair (i) predicational facts being grounded in plugging facts with (ii) instances being grounded in their slotites. And it’s natural to pair (i) plugging facts being grounded in instantiation facts with (ii) slotites being grounded in their instances. ‘Byzantine slotite theory’ endorses a third, unnatural, option, pairing (i) instantiation facts being grounded in plugging facts with (ii) the slotites being grounded in their instances.

Byzantine slotite theory isn’t just weird in that respect—it’s pointlessly over-complicated. Assume that bosons are fundamental and return to considering $b_1$’s spinning. Some fundamental fact, $\mathcal{U}$, is the ultimate explanans of that predicational fact. Since $b_1$ is fundamental, it’s natural to think it’ll be named in $\mathcal{U}$ and that it’s of the form $\Phi b_1...$ (where $\Phi$ is some fundamental, joint-carving predicate). If $\Phi$ is other than monadic, $\mathcal{U}$ is a relational fact about $b_1$. It can’t be a relation to a slotite, since byzantine slotite theory says slotites are derivative and thus can’t feature in
fundamental facts. Nor can it be a relation to a property for then—surely!—$\U$ would be a fact about $b_1$ instantiating the property; in that case, instantiation facts would be explaining predicational facts, not plugging facts (contrary to PLUGGING FIRST). Thus $\Phi$ must be a monadic predicate i.e. $\U$ is of the form $\Phi b_1$. $\U$ explains the fact that the relevant slotite of $Spin$ is plugged by $b_1$; call that fact $\mathcal{P}$. Given plugging facts are the explanation of predicational facts, $\mathcal{P}$ explains $\U$, where $\U$ is the predicational fact about how $b_1$ is, i.e. that $b_1$ spins. Here’s the rub: It’s weird for $\U$ to be a monadic fact about how $b_1$ is and not be the predicational fact that $b_1$ spins. If you’re going to have some monadic fact about $b_1$ be fundamental why not just make it the predicational fact you’re looking to explain in the first place? Why doesn’t $\U=\mathcal{P}$? Why are we going all around the houses drawing lines of explanation through plugging facts to get to $\U$? Byzantine slotite theory is simply a pointless complexification of nominalist slotite theory. It should be ignored.

8. Slotite Bundle Theory

8.1. Constructions of Co-plugged Slotites

On the flip side, slotites could be more fundamental than instances, presumably grounding the instance’s existence by being constituents of them. Just as with standard bundle theory, where instances are constituted by properties, we would have a theory whereby instances are constituted by slotites. Call this ‘slotite bundle theory’.

A standard bundle theorist would say of our bosons, $b_1$ and $b_2$, that, e.g., $b_1$ is a bundle of two co-instantiating properties ($Spin$ and $125 GeV^*$, being the property of having a mass of 125 GeV) whilst $b_2$ is a bundle of a different pair (e.g. $125 GeV^*$ and $Scalar$, being the property of not spinning). There’s a primitive of ‘co-instantiation’ which these properties stand-in in order to bundle themselves together and make up the instance. Slotite bundle theorists follow suit, but using a different primitive: ‘co-plugging’. Co-plugged slotites constitute an object.

Slotite bundle theorists will therefore think that a slotite possessed by $125 GeV^*$ and another possessed by $Spin$ are amongst $b_1$’s constituents. But they should also add that $b_1$ has amongst its constituents slotites possessed by relations. All slotite theorists hold that, for any $n$-adic relation $R$, such that $Rx_1...x_n$ is true, there are yoked slotites $\tau_1...\tau_n$ such that $\tau_n$ is plugged by $x_n$; the bundle slotite theorist is adding that $\tau_n$ is amongst the slotites which co-plug to comprise $x_n$. Given slotite theory, $\tau_1$ and $\tau_2$ are yoked slotites possessed by $Spinlike$, $b_1$ plugs $\tau_1$ whilst $b_2$ plugs $\tau_2$. Given slotite bundle theory, $\tau_1$ is a constituent of $b_1$ (and $\tau_2$ is a constituent of $b_2$). This has an interesting upshot, for now slotite bundle theory improves upon standard bundle theory. Standard bundle theorists analyse the instantiation of monadic properties by saying that such things are the constituents of instances, but bundle theorists are rather quieter when it comes to relations [cf Sider 2006: 395#4]. If relations aren’t constituents of bundles, bundle theory doesn’t provide a wholly general analysis of instantiation, for it no longer says anything about the (joint) instantiation of relations. If relations are constituents of bundles (and, e.g., $Spinlike$ is a constituent of $b_1$)—which is not what most bundle theorists believe—then, whilst we can now analyse $b_1$’s instantiating $Spinlike$, we can’t analyse its instantiating $Spinlike$ alongside $b_2$. Making relations constituents of instances only tells you that they’re a relatum of some instantiation of the relation, rather than telling us anything about which relata they are related to. Slotite bundle theory, taking a slotite analogue of the second approach, solves this problem (because the distinct slotites are possessed by distinct
relata and there’s a primitive yoking relation to tell you which are the relata of one and the same instantiation).

8.2 Tangent: A problematic alternative

This argument wouldn’t be as effective if the bundle theorist had an alternative solution to this problem of dealing with relations. One solution is to exclude relations from our ontology; if there aren’t any relations, the bundle theorist doesn’t have an issue. But throughout this paper I’ve assumed some version of slot theory is true and that’d be a pretty poor thing to assume if only monadic properties were possible (because slot theory only becomes useful/interesting once relations are introduced). So this response won’t work.

Hawthorne and Sider [2002: esp. 54-56] discuss an alternative response. Your bog standard bundle theorist treats co-instantiation as a plural predicate which pluralities of properties fall under (e.g. the plurality of Spin and 125 MeV are co-instantiated to constitute b1). Hawthorne and Sider take it up a notch: Co-instantiation can relate a varying number of pluralities of properties. Consider a relation of spatiotemporal separation (call it Distant) jointly instantiated by b1 (itself a bundle of Spin and 125 MeV) and b2 (a bundle of Scalar and 125 MeV). Hawthorne and Sider say: Distant is co-instantiated with (i) the plurality of Spin and 125 MeV and (ii) the plurality of Scalar and 125 MeV—call the state of affair of that being the case δ.

Hawthorne and Sider’s theory has problems that slotite bundle theory does not. Given their theory, co-instantiation must now be understood in one of two ways. It is either varigrade or it involves superpluralities; either option is problematic.

Consider the varigrade approach first. Standardly, we say that properties/relations have a fixed number of slots/places; perhaps the number of things which fill those slots varies, but not the number of slots themselves (e.g. Forming a circle has one slot but can vary over how many points collectively instantiate it). A relation is varigrade iff it can have a variable number of slots/places [MacBride 2005: 588]. Hawthorne and Sider might treat co-instantiation as a varigrade relation. In δ, co-instantiation has three slots (one for Distant, one for the plurality of Spin and 125 MeV, and one for the plurality of Scalar and 125 MeV). But in other states of affairs, it might have a different number of slots. For instance, the co-instantiation of b1’s constituents, Spin and 125 MeV, would be a case where co-instantiation had just one slot. So in some cases co-instantiation has three slots and in other cases it has one slot (and in yet more cases, it has a different number of slots). Co-instantiation is varigrade.

But varigrade properties come in for a hard time (e.g. from Armstrong [1978: 82-3] and Gilmore [2013: 229-31]). Like some others (e.g. MacBride [2005]) I’m more open to the idea of them, even at the fundamental level. For instance, a natural reading of endurantism makes parthood fundamentally varigrade. Mereological relations between spacetime regions, or between abstracta, are two-place e.g. one region is a part of another simpliciter and the number 2 is part of the real number series simpliciter. But when the mereological relation holds between enduring objects it is three-place e.g. the wheel is a part of the car at midday. If we think it’s the same relation in both cases, then the mereological relation will be varigrade. (Perdurantists probably say the same, but add that the temporally relativised relation is derivative of the fundamental dyadic relation, thus mereological relations aren’t fundamentally varigrade.) And it’s not just mereological relations. For instance, the same thinking applies to location relation. In some spacetimes, e.g. relativistic
spacetimes, location is a two-place relation, whilst in a spacetime with enduring spatial regions and separate temporal instants, spatial location will be a three-place relation between objects, spatial regions, and instants. If we think the same location relation holds in both spacetimes, location is varigrade.

But whilst I’m open to the idea of varigrade relations in general, co-instantiation seems nothing like these examples. In these examples, the adicity of the relation varies because it’s connecting things from wholly different ontological categories. Where the dyadic mereological relation relates spacetime regions or abstracta, the triadic relates objects and times (or spatial regions and times). Where the dyadic location relation relates objects to spacetime regions, the triadic location relation objects to enduring spatial regions and irreducible instants. If varigradicity is plausible, it’s plausible because in different cases there appears to be an entirely new ontological category of entities to be relating things too. And that’s not what’s going on with co-instantiation, where the extra places are added to make room for more of the same sort of thing (i.e. pluralities of properties). In the different cases where the adicity of co-instantiation varies it’s always relating properties, or pluralities of properties. It doesn’t seem like co-instantiation is a good candidate for a varigrade relation.

So consider the second approach. Co-instantiation isn’t varigrade. Instead it’s a monadic predicate with a single place that’s filled by a plurality. What’s contentious is that this plurality can itself include pluralities—that is, co-instantiation is a single place predicate filled by a ‘superplurality’ [Rayo 2006]. In the example above, co-instantiation is a monadic predicate holding of the superplurality of (i) Distant, (ii) the plurality of Spin and 125 MeV, and (iii) the plurality of Scalar and 125 MeV. But facts about co-instantiation are fundamental. So facts involving superplurals will have to be fundamental. And that’s the bit which I baulk at. Superplural facts might be derivatively true, but I’ve a hard time thinking that facts involving superplurals can be fundamentally true. (Indeed, we may have worries about whether facts involving just straightforward plurals can be fundamental, never mind superplurals [Sider 2011: 208-16].)5

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5 Where:

The xs are a plural-levelling of the ys iff all and only the singular entities amongst ys’s pluralities (or the pluralities amongst ys pluralities, or the pluralities amongst the pluralities amongst ys’s pluralities etc.) are amongst the xs.

ϕ is the levelling of ψ (with regards to the xs) iff (i) ϕ is a proposition of the form Φ(x₁,…xₙ…); (ii) ψ is a proposition of the form Φ(x₁,…xₙ…); (iii) the ys are a plural-levelling of the xs.

Hawthorne and Sider would also need:

LEVELLED DIFFERENCES: There are some xs such that F…xs… is true and yet a levelling of F…xs… (with regards to the xs) is false.

LEVELLED DIFFERENCES is necessary because the plurality of Distant, 125 MeV, Spin and Scalar (being the plural-levelling of the co-instantiating superplurality discussed in the main text) do not co-instantiate (for if they did, there’d be something which both spins and doesn’t spin). There may be cases of false levellings of true propositions at a non-fundamental level (consider Linnebo and Nicolai’s [2008: 193] ‘The square things and red things overlap’) but it seems harder to believe that there are such cases at the fundamental level.

Interestingly, if LEVELLED DIFFERENCES were true, the slotite theorist would—given a further radical commitment—be able to analyse ‘yoking’, achieving an ideologically lean theory analysing instantiation solely in terms of co-plugging. Given LEVELLED DIFFERENCES, constituency is nothing like mereological part-whole since pluralities of slotites could be constituents of things even though no slotite amongst that plurality was itself a constituent. Return to the bosons. τ₁ and τ₂ are yoked slotites of Spinlike, b₁ plugs τ₁ and b₂ plugs τ₂. We could now say that the plurality of τ₁ and τ₂ are constituents of b₁ even though neither slotite is a constituent of b₁. (We can also say the same of b₂) ‘Yoking’ can now be defined:

The τs are yoked =def ∃x [ x has the τs as constituents ∧ nothing amongst the τs is a constituent of x ]
In short, Hawthorne and Sider’s theory is not without its challenges.Slotite bundle theory, then, is an interesting alternative to consider.

8.3 Inter-depencence

Slotite bundle theory builds everything—instances, properties, slots, tropes—out of slotites. However, this ends up reintroducing mysterious connections. According to slotite bundle theory, slotites won’t be able to ‘float free’ of instances any more than standard bundle theory says similar of properties—if a slotite exists, it’s a constituent of something. Therefore, if one slotite of a relation exists, it’s a constituent of an instance and that instance must instantiate the relation. Necessarily, there must therefore be another (or many other) slotite(s) yoked to it. A mysterious connection has returned because the existence of some things is now necessitated by the existence of a distinct thing.

Above, I relieved necessary connections of their mystery by saying that one of the connected existents depended on the other. If relations have directions, e.g. if Taller than related the tall thing first and the shorter thing second, we could say that slotites plugged by the ‘later’ things depend on the slotite plugged by the ‘first’ thing. But this has two difficulties. First, Fine [2000] advances strong reasons to deny that relations have directions. Second, we’re left in the dark when it comes to symmetric relations, which are clearly directionless—given that prime candidates for fundamental relations are symmetric relations like spatiotemporal separation [Armstrong 1978: 90], that’s particularly problematic. So this approach is a possibility but probably unappealing.

The alternative is to say that no slotite is prior to a yoked compatriot. Instead, yoked slotites are inter-dependent: For any plurality of (maximally) yoked slotites, each slotite is wholly grounded in the remaining slotites. Not everything will inter-depend, since ‘unyoked’ slotites can still be fundamental, but this inter-dependence nevertheless extends to instances: if every instance is related to at least one other instance, every instance inter-depends upon some other instance (the view of Madhyamika Buddhism); if any two instances are related, all instances inter-depends (the view of Huayan Buddhism [Bliss and Priest 2017]).

Inter-dependence is contentious since it requires dependence to be non-symmetric (and/or grounding, ontological priority, or whatever relation your metametaphysics favours). It’s not been popular in contemporary analytic metaphysics, which has inclined to thinking dependence is well-founded. But inter-dependence has not gone without defence [Barnes 2018; Thompson 2016, 2018]. And whilst I’m in no rush to accept inter-dependent entities, I’m less hostile to them than other philosophers—after all, they didn’t seem unintuitive to Buddhist philosophers and that fact alone should give one pause to too fervently thinking that dependence is well-founded. Also, it’s particularly defensible in this case because, if you have any time whatsoever for the possibility of inter-dependent entities, yoked slotites are surely the best candidates for being inter-dependent. If you want to go inter-dependent about anything, go inter-dependent about these things.

The plurality of τ₁ and τ₂ are yoked because something (in this case, two things—b₁ and b₂) has the plurality as a constituent even though τ₁ isn’t a constituent of it and τ₂ isn’t a constituent of it either. Then say:

x plugs slotite τ₁ =_y b₁ x is a bundle of co-plugged slotites, the yₙ, and either (a) τ₁ is a slotite amongst the yₙ or (b) τ is a slotite amongst some plurality that is amongst the yₙ.
Elementary reflection reveals the second radical commitment that we would need: This only works if every relation is symmetric. But that is not an entirely unheard of commitment [Dorr 2004]. If you want ideological parsimony, this is a possible way forward.
However, it does undermine some of the dialectical force for slotite theory in the first place. If non-symmetric dependence relations are on the table then the original objection to instantiation relative slot theory (back in §2) loses its bite, for it played on the idea that dependence was asymmetric.

9. Summary

This paper hasn’t motivated slot theory, nor defended any particular brand of it. I’ve merely surveyed logical space, mapping out where one would find oneself were one motivated to become a slot theorist. Here’s a pictorial summary:

I leave with some parting comments on our options. For those who endorse Predication First, slot theory will work just fine—just endorse instantiation relative slot theory and be done with it. (And, if you want slotities too, endorse nominalist slotite theory.) We should not endorse Instantiation First; filling facts are more fine grained than instantiation facts and so Filling First will be preferable. But those who believe Filling First are faced by POF. I followed through the solutions until I arrived at slotite theory, which comes with a commitment to metaphysically inter-dependent entities. For those who are staunchly committed to asymmetric dependence relations, I suspect that spells bad news for slotite bundle theory a fortiori bad news for Filling First—indeed, one possible conclusion you may draw from this paper is that slot theorists should believe Predication First.

10. Bibliography

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